Sovereign Debt and CDS - A Welfare Analysis

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Abstract

We analyze a model of government borrowing, where the lender can insure himself against government default by signing a contract with a third party. Under quite general specifications we characterize the sub-game perfect equilibrium and compare it to the second-best and an economy where no such insurance is available. We find that under risk-neutrality of all parties the lender always chooses the efficient level of credit insurance and credit insurance is thus welfare improving compared to an economy without credit insurance. This is however no longer true in the case of risk-aversion of the government. We provide precise conditions under which an economy with credit insurance is strictly pareto-inferior compared to an economy without credit insurance.

1 Introduction

Credit Default Swaps have been at the center of the discussion since the beginning of the recent financial crisis. Their impact on the market for sovereign debt has been highly debated. Most recently, during the Greek debt renegotiation, it was frequently argued in the press that a number of hedge funds who invested in CDS hindered the debt renegotiation process and thus made an efficient resolution of the Greek debt problem more difficult. While from an ex-post perspective hindering renegotiation is clearly welfare decreasing, it is not obvious whether this is also true from an ex-ante perspective. This work aims at analyzing the welfare implications of credit insurance on the renegotiation process for government debt in an analytical framework.

We analyze a model of government borrowing, where the lender can insure himself against government default by signing a contract with a third party. Under quite general specifications we characterize the sub-game perfect equilibrium and compare it to the second-best and an economy where no such insurance is available. As commonly assumed in the literature for government debt (e.g. Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2007)) we allow for risk-averse preferences of the government, which enables us to study the impact of credit insurance on consumption smoothing. Previous work on this subject has focused on a risk-neutral borrower and was therefore not able to address this issue. A second difference to the existing literature in this field is that the level of borrowing is a decision variable, while in previous studies it has been taken as exogenous.

We find that by investing in credit insurance the lender can strengthen its outside option during renegotiation and consequently obtain a higher share of the bargaining surplus compared to when no such option is available. Even though credit insurance never pays in equilibrium, it enables the lender to enforce a higher debt repayment. Credit insurance thus works as a commitment device. In case of a risk neutral government this is ex-ante welfare improving: enforcing a higher amount of repayment leads to improved borrowing conditions and thus relaxes the borrowing constraint of the government. We also specify the assumption on the competitive structure of the economy that are needed for this mechanism to go through.

The government is however not the party who chooses the level of credit insurance. Under the assumption of risk-aversion the lender no longer always chooses the socially efficient level of credit insurance and credit insurance may even decrease welfare. The reason for this is that credit insurance has two opposing effects in this case. On one hand, by increasing the amount of borrowing available, credit insurance helps the government to transfer wealth between periods and therefore facilitates consumption smoothing across time. On the other hand renegotiation helps to add contingency to the otherwise non-contingent bond. By enforcing a higher repayment in the low state, credit insurance reduces this benefit and as a result makes consumption smoothing between states more difficult. Which effect prevails depends on the endowment in the low state. If the endowment in the low state is very low compared to the initial wealth and the endowment in the high state, the second effect is stronger so that credit insurance is welfare decreasing. This is because the marginal utility is decreasing in case of risk-averse preferences, and thus for a low endowment in the low state compared to the level of consumption in period 1, the government values the additional consumption in the first period less compared to the higher repayment in the low state. The previous literature in this field has not been able to address these opposing welfare effects, as the borrower is always assumed to be risk-neutral in these models. The contribution of this paper is thus to develop a model that is general enough to capture these opposing effects of credit insurance has on the welfare due to risk-aversion.

The work most closely related to the present is Sambalaibat (2011) who studies the impact of credit insurance on the moral hazard problem of the government. In her model investment is not observable and renegotiation is assumed to happen only in the low state. As a consequence the government does not fully take the losses in the low state and therefore has an incentive to invest less than the socially efficient amount. By enforcing a higher repayment in the low state, credit insurance ameliorates this moral hazard problem. We confirm that under risk-neutrality credit insurance has an welfare improving effect even in an environment of full information. Bolton and Oehmke (2011) study the impact of credit insurance in a corporate debt model with an exogenous amount of investment needed. They also find that credit insurance works as a commitment device and thus relaxes the borrowing constraint of the firm. This makes it possible to finance projects that were otherwise not possible to realize, which is welfare improving. In equilibrium however, they find that for high levels of borrowing, the lender may choose a higher level of credit insurance than what would be socially efficient as it leads to inefficient default in some states. We also find a similar result. In our model however the debt level is an endogenous choice of the government. This enables us to show that in case of risk-neutrality, the government never finds it optimal to choose such high debt levels that lead to inefficient default, so that there is no over-insurance in equilibrium.

This work is also related to Grossman and Van Huyck (1988) and Kehoe and Levine (2006) who show that default can add contingency to the otherwise non-contingent bond contract. In our work we show that credit insurance hinders this welfare improving aspect of default or renegotiation and thus might also be welfare decreasing.

2 Model

Suppose there is a government which can access the international credit markets either for consumption smoothing purposes and/or to invest into a productive technology. We make the model general enough to account for both, later we analyze the impact of credit insurance on investment and consumption smoothing separately. The economy lasts for two periods: In period 0 the government can borrow from one of the risk-neutral lenders a notional amount of debt b at a price $q(b)$. We make the standard assumption that while credit markets are competitive ex-ante, the government can only borrow from one of the lenders. The government is also endowed with some initial wealth w_0, which is assumed to be low enough so that the government wants to borrow. It can use its initial wealth and the proceeds from borrowing to either consume in period 0 or to invest into its productive technology. k units invested in period 0 produce $f(\theta, k)$ units of the consumption good in period 1, where f is increasing in both arguments and concave with respect to the second argument. The productivity factor θ is a random variable which can take two values: θ^H with probability π and θ^L with probability $1 - \pi$. The realization of θ is observable to both parties but not contractible, so that the amount of debt cannot be made contingent on the state. The government values consumption in period t according to a utility function $u(c_t)$. We first make some general observations based on u being continuous, increasing and twice differentiable, then we restrict the analysis further to the case of risk-neutrality and risk-aversion.

There is limited commitment on the side of the government so that it may decide not to repay its debt. In this case it suffers a loss of $\lambda \in [0, 1]$ to its output. This loss can be interpreted as corresponding to the cost of market exclusion in an infinite horizon economy. Default thus leads to an ex-post efficiency loss. This opens up the room for renegotiation: the government and lender can come together and renegotiate the amount of outstanding debt. Renegotiation is costly however, at a smaller cost $\delta < \lambda$ to total output. The surplus from renegotiation is shared according to a Nash bargaining rule.

The lender has also the option to enter into an insurance contract with a third party, a risk-neutral insurer. This contract, in practice called credit default swap, pays a mutually agreed amount of $i \geq 0$ in case of a full default of the government for a premium $q^{CDS}(i)$. Similar to the bond market, the insurance market is perfectly competitive ex-ante, but the lender can only contract with one single insurer. As standard in the literature (e.g. Bolton and Oehmke (2011), Sambalaibat (2011)) renegotiation is considered voluntary, so that credit insurance does not pay after successful renegotiation. None of the agents discount the future.

Timing is as follows: At the beginning of period $t = 0$ the government simultaneously chooses the amount of investment k and the notional amount of debt b. In the middle of period $t = 0$, after having observed the level of investment, lenders give a quote at which price $q(b)$ they are willing to take on the full notional b. Then the government decides on which offer to accept or whether to reject all. If it accepts one offer it receives an amount $q(b)\,b$ of lending. At the end of period $t = 0$ insurers quote a price schedule to the lender at which they are willing to pay a notional amount i in case of default. The lender then decides on the level of credit insurance. At the beginning of period $t = 1$ the productivity shock θ is realized. After observing the shock the government and the lender can decide whether they want to enter into renegotiation. If renegotiation is rejected by one of the parties, the government decides on whether to fully repay or fully default. We make thus the assumption that renegotiation happens before the actual repayment decision of the government. This insures that the government can only renegotiate in states where it actually would default if there was no renegotiation in place.[1] The status quo of the bargaining game is thus state contingent and determined by the repayment decision of he government.

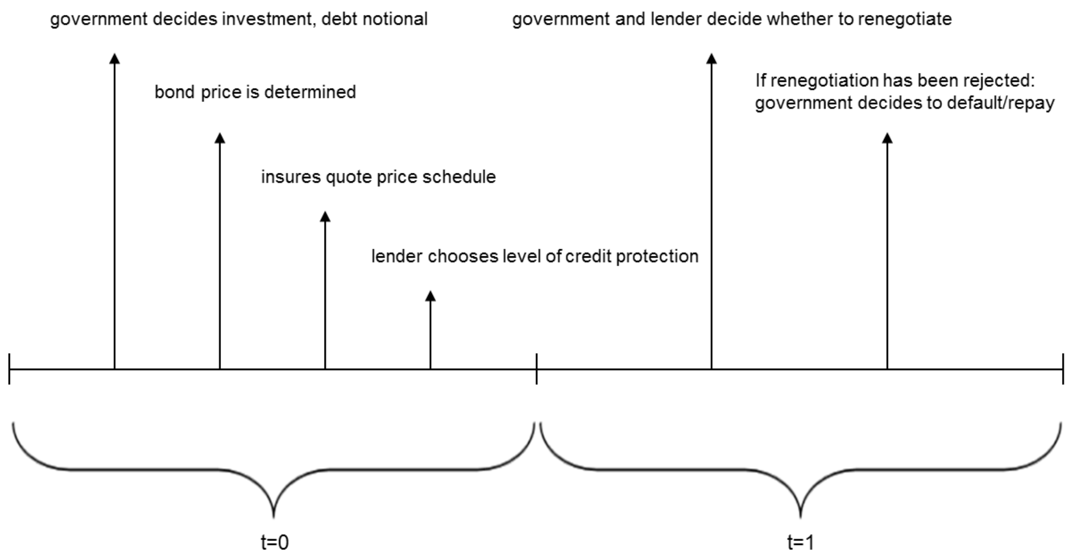

Figure 2.1: timing

[1]If we would assume that renegotiation happens after the default decision of the government, this would imply that the government can claim opportunistically default in some states where it would fully repay if there was no renegotiation procedure in place as it knows to get a better deal during renegotiation.

3 Equilibrium

In this section we describe the equilibrium with and without CDS. The next section will characterize the equilibrium. We describe the decision problem of each agent at each decision node.

3.1 The government's problem in period 0

The government chooses the level of debt b and investment k at the beginning of period 0 correctly anticipating bond prices, in order to maximize lifetime consumption

$$u\left(c_0\right)+\pi u\left(c_H\right)+(1-\pi)\, u\left(c_L\right)$$

where c_0 denotes consumption in period 0 and is given by the initial wealth plus the proceeds from issuing debt minus the amount invested into the productive technology

$$c_0=w_0+q(b,k)b-k$$

. c_s denotes consumption in period 1 in state s and depends on whether both parties agree to renegotiation in period 1 and in case they do not, whether the government prefers to fully repay or default. In case of renegotiation the government's consumption in state s is given by

$$c_s^{ren}=(1-\delta)\, f\left(\theta^s,k\right)-d^{ren}\left(i,\theta^s,k\right)$$

as the government suffers a loss of δ to output and repays the renegotiated amount of debt $d^{ren}(i, \theta^s, k)$ determined by Nash bargaining in period 1. In case of full repayment the government consumes

$$c_s^{pay} = f(\theta^s, k) - b$$

as it suffers no loss to output but repays the full outstanding debt. Default leads to an output loss of λ while there is no repayment so that consumption in this case is given by

$$c_s^{def} = (1 - \lambda) f(\theta^s, k)$$

3.2 Bond price

In the middle of period 0, after having observed the governments investment k and the amount of debt b the government wants to borrow, lenders quote the price at which they are willing to take on the full amount of debt, correctly anticipating the choice of credit insurance and the repayment of debt. Its decision problem is thus given by

$$q(b, k) \in \arg\max_q -qb + \mathbb{E}_\theta \left[d^{eft}\left(i^*(k, b), \theta, k, b\right) \right] \tag{3.1}$$

where $d^{eft}(i, \theta, k, b)$ is the effective repayment of debt in period 1 and $i^*(k, b)$ is the optimal choice of credit insurance by lender as described next. We will see in section 4, that in equilibrium competition amongst lenders will lead to zero profits.

3.3 The lender's choice of CDS

At the end of period 0 the investor chooses the optimal level of credit insurance taking into account the effect its choice has on the bargaining outcome in the following period. In order to obtain credit insurance with a notional of i, the lender has to pay a premium $q^{CDS}(i,k,b)\,i$ to the insurer, where $q^{CDS}(i,k,b)$ is the price schedule quoted by the insurer. In period 1 the lender receives the effective repayment of debt $d^{eft}(i,\theta,k,b)$ depending on whether there is default, renegotiation or repayment. Only if the government defaults so that the effective repayment equals to zero, does the credit insurance pay out the promised amount i to the lenders. The lenders profit maximization problem is thus given by

$$i^*(k,b) \in \max_i \; -q^{CDS}(i,k,b)\,i + \mathbb{E}_\theta\left[d^{eft}(i,\theta,k,b) + I_{\left\{d^{eft}(i,\theta,k,b)=0\right\}} i\right] \tag{3.2}$$

where I denotes the indicator function[2].

3.4 Renegotiation

At the beginning of period 1 both parties must decide whether they are willing to renegotiate the bond contract. If renegotiation takes place the renegotiated amount of

[2]Note that since the amount of lending payed to the government has been already determined at the beginning of period $t = 0$ it is considered as sunk and therefore not taken into account in its decision problem at this point.

debt is determined according to Nash bargaining and thus solves the following problem

$$d^{ren}(i,\theta^s,k) = \arg\max_d \left[u\left((1-\delta) f(\theta^s,k) - d\right) - u\left(c^{quo}(\theta^s,k)\right)\right]\left[d - d^{quo}(\theta^s,k)\right] \tag{3.3}$$

Where $c^{quo}(\theta^s,k)$ is the amount of consumption the government would receive in the state-contingent status quo, which depends on the repayment decision of the government at the end of period 1 as described in the next section. If renegotiation is successful the government suffers a cost of $(1-\delta)$ to its output and needs to repay the renegotiated amount. The welfare improvement for the government from renegotiation is thus given by $u((1-\delta)f(\theta^s,k)-d) - u(c^{quo}(\theta^s,k))$. The lender on the other hand receives and amount of d if renegotiation is successful, while $d^{quo}(\theta^s,k)$ is the amount of repayment he would receive either from the government or its insurance contract if renegotiation was not successful. In order to make both parties to agree into renegotiation, they must prefer the renegotiation outcome to the status quo.

3.5 Status Quo

In case renegotiation has been rejected by either the government or the lender, the government can choose only between full repayment or full default. The government will choose to repay, if the contractual amount of debt is smaller than the cost of default

$$f(\theta^s,k) - b \geq (1-\lambda) f(\theta^s,k) \tag{3.4}$$

otherwise it will prefer to default.

3.6 Equilibrium Definition and Second Best

The equilibrium concept we use is sub-game perfect equilibrium. The next section characterizes the equilibrium. We then compare the equilibrium to the second-best where a social planner can choose the amount of credit insurance before the economy starts and the economy evolves otherwise as previously described. Since lenders and insurers make zero profits in equilibrium this is equivalent to a situation where the government chooses the level of credit insurance. We also compare the equilibrium to a situation where the lender cannot insure itself against default so that the equilibrium level of credit insurance $i^*(k,b)$ is exogenously set to zero.

4 Characterization

We now characterize the sub-game perfect equilibrium with credit insurance by backwards induction starting from the final decision node. In case renegotiation is rejected the government needs to decide of whether to repay fully or default. Rewriting condition (3.4) we can see that the government repays if debt is small compared to output

$$b \leq \lambda f(\theta^s, k)$$

. The state-dependent status quo for the government in the renegotiation game can thus be written as

$$c^{quo}\left(\theta^{s},k\right)=\begin{cases}f\left(\theta^{s},k\right)-b & b\leq\lambda f\left(\theta^{s},k\right)\\ \left(1-\lambda\right)f\left(\theta^{s},k\right) & b>\lambda f\left(\theta^{s},k\right)\end{cases}$$

The status quo for the lender on the other hand is given by

$$d^{quo}\left(\theta^{s},k\right)=\begin{cases}b & b\leq\lambda f\left(\theta^{s},k\right)\\ i & b>\lambda f\left(\theta^{s},k\right)\end{cases}$$

as in case of a full default credit insurance pays out the contractual amount of i, while in case of a full repayment the lender receives the contractual amount of debt b. Renegotiation can only take place if both parties can be made better off than these levels. We can see immediately that in states such that there is full repayment there is no room for renegotiation. This is because the government would only agree into renegotiation if the renegotiated amount of debt was strictly below the contractual amount as it also suffers the cost of renegotiation. On the other hand the lender would not agree to such a deal, as he would receive a full repayment of the contractual amount of debt if he would reject renegotiation. Thus, there can be no efficiency improvement for states where there is full repayment and renegotiation therefore does not take place. If on the other hand debt levels are high enough $b>\lambda f\left(\theta^{s},k\right)$ such that if renegotiation was rejected there would be default, renegotiation can lead to an ex-post efficiency improvement as it is less costly than full default. If however the level of credit insurance is higher than the full surplus from renegotiation $(\lambda - \delta) f(\theta^s, k)$ it is not possible to make the lender to agree and thus also in this case renegotiation is rejected and followed by full default. The next lemma summarized the effective repayment of debt.

Lemma 1. *In states where the debt-to-output ratio is low so that* $b \leq \lambda f(\theta^s, k)$ *there is full repayment of debt. When* $b > \lambda f(\theta^s, k)$ *and the level of credit insurance is low enough, so that* $i \leq (\lambda - \delta) f(\theta^s, k)$ *renegotiation takes place. If however* $b > \lambda f(\theta^s, k)$ *and* $i > (\lambda - \delta) f(\theta^s, k)$ *renegotiation is rejected and the government defaults.*

The effective amount of repayment can thus be written as

$$d^{eft}(i, \theta^s, k, b) = \begin{cases} b & b \leq \lambda f(\theta^s, k) \\ d^{ren}(i, \theta^s, k) & b > \lambda f(\theta^s, k), \ i \leq (\lambda - \delta) f(\theta^s, k) \\ 0 & b > \lambda f(\theta^s, k), \ i > (\lambda - \delta) f(\theta^s, k) \end{cases} \tag{4.1}$$

We can thus see that the level of credit insurance only matters for high enough levels of debt. Before proceeding to the choice of credit insurance of the lender we make an observation regarding the relation between the renegotiated amount of debt and credit insurance. We find the intuitive result that whenever there is renegotiation in period 2, the renegotiated amount of debt is increasing in the level of credit protection.

Lemma 2. *Whenever renegotiation is accepted the renegotiated amount of debt repayment* $d^{ren}(i, \theta^s, k)$ *is non-decreasing in the amount of credit insurance* i*.*

Proof. see appendix A.1 □

We now proceed to analyze the optimal choice of credit insurance by the lender. Competition among risk-neutral insurers implies that the price of credit insurance equals to the probability of default for each level of credit insurance chosen

$$\begin{aligned} q^{CDS}(i,k,b) &= P\left(\left\{d^{eft}(i,\theta,k,b)=0\right\}\right) \\ &= \mathbb{E}_\theta\left[I_{\left\{d^{eft}(i,\theta,k,b)=0\right\}}\right] \end{aligned}$$

so that the problem of the lender (3.2) simplifies to

$$i^*(k,b) \in \max_i \mathbb{E}_\theta\left[d^{eft}(i,\theta,k,b)\right] \tag{4.2}$$

From this expression we can see that the benefit of credit insurance to the lender comes purely from strengthening its bargaining power when there is renegotiation. This is because the lender internalizes that the price of credit insurance cancels out with its expected payment in case of default. Additionally, however, credit insurance also leads to a higher repayment to the lender in case of renegotiation. Credit insurance thus imposes a non-pecuniary externality on the bargaining process between the government and the lender. As we have seen in lemma 2 the renegotiated amount of debt is increasing with the level of credit insurance, so that the effective repayment of debt in each state is maximized by choosing an amount of credit insurance that equals the full bargaining surplus, $i=(\lambda-\delta)f(\theta^s,k)$. With this choice the lender is able to extract the full surplus from renegotiation[3]. Credit insurance thus effectively changes the

[3] Note that this observation is in fact not particular to Nash bargaining, as the lender is able to extract the full bargaining surplus under any bargaining protocol by choosing $i=(\lambda-\delta)f(\theta^s,k)$ as this choice pushes the government exactly to its participation constraint.

bargaining power between the government and the lender during renegotiation. Note however that the lender needs to choose the level of credit insurance before the uncertainty about θ has been resolved. Choosing $i = (\lambda - \delta) f\left(\theta^H, k\right)$ would lead to full surplus extraction in the high state but to default in the low state. It might be preferable to choose a level of credit insurance of $i \leq (\lambda - \delta) f\left(\theta^L, k\right)$ in order to allow for renegotiation in the low state, even though this implies that less than the full surplus is extracted in the high state. The following proposition characterizes the optimal choice of the lender:

Lemma 3. *For low levels of debt such that* $b \leq \lambda f\left(\theta^L, k\right)$ *the amount of credit insurance is irrelevant, so in particular setting* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$ *is weakly optimal for the lender. For intermediate levels of the debt such that*

$b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ *the optimal choice of credit insurance is* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$. *For high levels of debt* $b > \lambda f\left(\theta^H, k\right)$ *the optimal choice of credit insurance is either* $i^*(k, b) = (\lambda - \delta) f\left(\theta^L, k\right)$ *or* $i^*(k, b) = (\lambda - \delta) f\left(\theta^H, k\right)$.

Proof. From Lemma 1 it follows that there is full repayment of the contractual amount of debt in both states when $b \leq \lambda f\left(\theta^L, k\right)$, so that credit insurance has no impact on the debt repayment. The price of credit protection is also zero, as credit insurance does not pay out in neither of the states. Any level of credit insurance thus gives the same profit to the lender. For $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ there is full repayment of debt in the high state, but either default or renegotiation in the low state, depending on the level of credit protection chosen by the lender. If he chooses $i \leq (\lambda - \delta) f\left(\theta^L, k\right)$ there is renegotiation in the low state so that the expected repayment to the lender is

$$\pi b + (1 - \pi)\, d^{ren}\left(i, \theta^L, k\right)$$

while if he chooses $i > (\lambda - \delta)\, f\left(\theta^L, k\right)$ there is default in the low state so that his expected payoff is

$$\pi b$$

Clearly the first option is more profitable for the lender and since the amount of renegotiated debt $d^{ren}\left(i, \theta^L, k\right)$ is increasing with the level of credit insurance (Proposition 2) the optimal choice is $i^*(k, b) = (\lambda - \delta)\, f\left(\theta^L, k\right)$ in this case. For $b > \lambda f\left(\theta^H, k\right)$ there is either default or renegotiation in both states. If the lender chooses $i \leq (\lambda - \delta)\, f\left(\theta^L, k\right)$ there is renegotiation in both states and the expected payoff to the lender is

$$\pi d^{ren}\left(i, \theta^H, k\right) + (1 - \pi)\, d^{ren}\left(i, \theta^L, k\right)$$

while if he chooses $i \in \left((\lambda - \delta)\, f\left(\theta^L, k\right), (\lambda - \delta)\, f\left(\theta^H, k\right)\right]$ there is default in the low state so that his expected payoff is

$$\pi d^{ren}\left(i, \theta^H, k\right)$$

. Note however that the lender is able to extract the full surplus in the high state if he chooses $i = (\lambda - \delta)\, f\left(\theta^H, k\right)$, which might be more profitable than choosing $i = (\lambda - \delta)\, f\left(\theta^L, k\right)$ in order to allow for renegotiation in the low state. Clearly, he would never choose any level in between, since it would still lead to full default in the low state, while extracting less than the full surplus in the high state. Which option is more profitable depends on the probability π as well as the relative difference of the shocks. Choosing an amount of credit insurance $i > (\lambda - \delta) f\left(\theta^H, k\right)$ on the other hand would lead to default in both states and thus an expected repayment of 0, which can never be optimal. $\square$

Note that by choosing $i^*(k,b) = (\lambda - \delta) f\left(\theta^H, k\right)$ the lender forces the government into default in the low state. This results in a potential an ex-ante efficiency loss as lenders and insurers make zero profit and the government suffers the higher cost of default compared to renegotiation. We will see in the next section that under risk-neutrality this efficiency loss never arises in equilibrium, as the government never chooses a debt level of $b > \lambda f\left(\theta^H, k\right)$ which would make this choice of credit insurance optimal for the lender.

We next derive the price of debt. Competition in the market for bonds implies that the lenders profit given by equation (3.1) must equal to zero. The price of debt $q(b,k)$ is thus given by the following equation

$$q(b,k)b \quad = \quad \mathbb{E}_\theta\left[d^{eft}\left(i^*(k,b), \theta, k, b\right)\right] \tag{4.3}$$

The value of debt must equal its expected repayment. The government when choosing the amount of debt b and investment k, correctly anticipates the bond prices quoted by the lenders and the effective repayment in period 1.We now analyze the governments problem for the case of risk-neutrality and risk-aversion separately and study the implications credit insurance on welfare for each case in turn. We will see that the effect of credit default swaps can work in different directions.

5 Risk-Neutrality

5.1 Equilibrium

The government anticipates that by borrowing less than $\lambda f\left(\theta^L, k\right)$ it will fully repay in both states and the bond prices therefore equals to 1. The maximal welfare it can achieve by borrowing such an amount is thus given by

$$\begin{aligned} \underline{W}^{CDS} &= \max_{b,k} \quad w_0 - k + b + \pi\left[f\left(\theta^H, k\right) - b\right] + (1-\pi)\left[f\left(\theta^L, k\right) - b\right] \\ s.t. &\qquad b \leq \lambda f\left(\theta^L, k\right) \\ &\qquad w_0 - k + b \geq 0 \end{aligned}$$

where the second constraint comes from the non-negativity of consumption requirement in period 0 (in period 1 it is automatically satisfied by the constraint on debt). Risk-neutrality implies that the proceeds from issuing debt in period 0 cancels out exactly with the expected repayment in period 1, the problem thus simplifies to

$$\underline{W}^{CDS} = \max_{b,k} \quad w_0 - k + \pi f\left(\theta^H, k\right) + (1-\pi) f\left(\theta^L, k\right)$$

$$\text{s.t.} \qquad b \le \lambda f\left(\theta^L, k\right)$$

$$k \le w_0 + b$$

Borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ on the other hand leads to renegotiation in the low state but full repayment in the high state. Already canceling out the proceeds from issuing debt and its expected repayment, the welfare the government can obtain with this choice is thus given by

$$\overline{W}^{CDS} = \max_{b,k} \quad w_0 - k + \pi f\left(\theta^H, k\right) + (1-\pi)(1-\delta) f\left(\theta^L, k\right)$$

$$\text{s.t.} \qquad b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$$

$$k \le w_0 + \pi b + (1-\pi) d^{ren}\left(i^*(k,b), \theta^L, k\right) \tag{5.1}$$

as the government needs to pay the cost of renegotiation in the low state, but also repays only the renegotiated amount of debt which is anticipated by the lender and thus affects the borrowing constraint. Comparing the two problems we can see that if wealth is high enough such that the government can invest the efficient level of capital k^{**} by borrowing less than $\lambda f\left(\theta^L, k^{**}\right)$ it will always do so. If however the optimal investment is much higher than what it can achieve by borrowing $\lambda f\left(\theta^L, k\right)$ it might prefer to incur the cost of renegotiation in the low state in return for relaxing the borrowing constraint. The following lemma shows that the government never finds it optimal to borrow an amount $b > \lambda f\left(\theta^H, k\right)$ as this would lead to renegotiation in both states.

Lemma 4. *For any level of credit insurance, the government never finds it optimal to borrow strictly more than $\lambda f\left(\theta^H, k^*\right)$ where k^* denotes the optimal level of investment in equilibrium.*

Proof. Suppose not and it would be optimal to borrow amount b^* strictly higher than $\lambda f\left(\theta^H, k^*\right)$. This would result in renegotiation in the high and in the low state. Now consider the alternative choice $\tilde{b} = \lambda f\left(\theta^H, k^*\right)$. This choice implies that there is full repayment in the high state and renegotiation in the low state. Note that the level of debt only appears in the constraint on investment. Under the original choice b^* the constraint reads $\pi d^{ren}\left(i, \theta^H, k^*\right) + (1-\pi)\, d^{ren}\left(i, \theta^L, k^*\right)$ while under the alternative choice $\tilde{b}$ it is given by $\pi\lambda f\left(\theta^H, k^*\right) + (1-\pi)\, d^{ren}\left(i, \theta^L, k^*\right)$. Since the latter is larger for arbitrary i, the borrowing constraint is more slack when borrowing $\tilde{b}$ than under the original level of debt. Also the government does not suffer the cost of renegotiation δ in the high state under $\tilde{b}$. Welfare is thus strictly higher under the alternative choice of debt, the original choice can thus not have been optimal. □

5.2 Second-best

Let us now compare the equilibrium with the second-best. In the second-best the planner chooses the amount of credit insurance instead of the lender. Since for low levels equilibrium values of debt credit insurance does not have an impact on the repayment decision, the planner cannot improve on the equilibrium amount of welfare. If on the other hand the government chooses an amount of borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ the planner could potentially improve on welfare by further relaxing the constraint (5.1) as he chooses the amount of credit insurance i directly which determines the bargaining outcome. This might lead to an efficiency improvement as the lender is less constraint on its investment decision. However, as derived in proposition (3.2) the lenders chooses a level of credit insurance of $i^*(k,b) = (\lambda - \delta) f\left(\theta^L, k\right)$ for intermediate levels of debt, so that he extracts the full surplus from renegotiation. The constraint thus cannot be further relaxed by a social planner.

The other inefficiency that might potentially arise in equilibrium is that the lender chooses such a high level of credit insurance that the government is forced into default in the low state and thus suffers the higher cost of default while renegotiation would have been possible for a lower level of credit insurance. In lemma 3.2 we found that this might occur for high levels of debt $b > \lambda f(\theta^s, k)$. Lemma 4 however shows that such a high amount of debt is never chosen in equilibrium, so that this kind of inefficiency never occurs in equilibrium. We summarize the finding in the following proposition

Proposition 5. *Under risk-neutrality the lender chooses the socially-efficient level of credit insurance.*

Since the planner could have chosen an amount of credit insurance which equals to zero but did instead choose the equilibrium level, it is an immediate consequence of the previous proposition that the equilibrium with credit insurance pareto-dominates the equilibrium without credit insurance.

Corollary 6. *The welfare in the economy with credit insurance is higher than in the* *economy without CDS.*

6 Risk-Aversion

We now proceed to analyze the welfare properties of the equilibrium when the government has risk-averse preferences

$$u(c)$$

with u strictly concave and the usual Inada condition

$$\lim_{c \to 0} u'(c) = \infty$$

applies. For the ease of exposition we will only consider pure endowment economies such that the production function becomes $f(\theta, k) = \theta$ from now on.[4] This setup is frequently used in the sovereign debt literature and therefore of particular interest. We will first show on hand of an example why the level of credit insurance chosen by the lender may no longer be constraint efficient. Then we derive the more general result that under risk-aversion the lender generally (weakly) over-insures. We then proceed to compare the economy with credit insurance to the economy without credit insurance. We first provide some intuition on the hand of two examples on what are the trade-offs between the two scenarios. We then give sufficient conditions under which the economy without credit insurance strictly pareto dominates and as a consequence the economy

[4]Clearly, under the assumption of risk-neutrality, if the output is independent of investment and only depends on the shock θ (as in a pure endowment economy) credit insurance does not have any effect. In the this section we will thus look at the more interesting case of a pure endowment economy with risk aversion.

with credit insurance is inefficient.

6.1 Equilibrium

As in the previous section, the governments problem depends on the level of debt. For low levels of debt $b \leq \lambda f\left(\theta^L, k\right)$ there is full repayment in both states and the government's problem becomes now

$$\begin{aligned} \underline{W}^{CDS} &= \max_b \quad u\left(w_0+b\right)+\pi u\left(\theta^H-b\right)+\left(1-\pi\right)u\left(\theta^L-b\right) \\ s.t. & \qquad b \leq \lambda\theta^L \end{aligned}$$

. The Inada condition implies that consumption in period 0 is positive, so we no longer need the extra constraint as in the case of risk-neutrality. By borrowing $b \in \left(\lambda f\left(\theta^L, k\right), \lambda f\left(\theta^H, k\right)\right]$ on the other hand we have renegotiation in the low state but full repayment in the high state. So that government's problem is

$$\begin{aligned} \overline{W}^{CDS} &= \max_b \quad u\left(w_0+\pi b+\left(1-\pi\right)\left(\lambda-\delta\right)\theta^L\right)+\pi u\left(\theta^H-b\right)+\left(1-\pi\right)u\left(\left(1-\lambda\right)\theta^L\right) \\ s.t. & \qquad b \in \left(\lambda\theta^L, \lambda\theta^H\right] \end{aligned} \tag{6.1}$$

Similarly as in the case of risk neutrality we can show that the government never finds it optimal to choose and amount of borrowing above $\lambda\theta^H$. However we need to make an extra assumption.

Assumption (A1)

$$\theta^H > \frac{2\lambda}{\lambda-\delta}\theta^L$$

This assumption ensures that the renegotiated amount of debt $d^{ren}\left(i,\theta^H\right)$ is always larger than $\lambda\theta^L$ (under any level of credit insurance), so that borrowing $d^{ren}\left(i,\theta^H\right)$ leads to renegotiation or default in the low state depending on the level of credit insurance.[5]

Lemma 7. *Under any level of credit protection, the government never finds it optimal to borrow strictly more than* $\lambda\theta^H$

Proof. Suppose not and it would be optimal to borrow amount b^* strictly higher than $\lambda\theta^H$. In what follows we will show that by choosing an alternative debt level of $\tilde{b} = d^{ren}\left(i,\theta^H\right) < \lambda\theta^H$ the government can do at least as good. We distinguish different cases depending on the level of credit insurance:

case 1 $i \leq \lambda\theta^L$: as we have seen in section 4 under $b^* > \lambda\theta^H$ this implies that there is renegotiation in both states

$$c_0 = w_0 + \pi d^{ren}\left(i,\theta^H\right) + (1-\pi)\, d^{ren}\left(i,\theta^L\right) \quad c_H = (1-\delta)\,\theta^H - d^{ren}\left(i,\theta^H\right)$$
$$c_L = (1-\delta)\,\theta^L - d^{ren}\left(i,\theta^L\right)$$

while under $\tilde{b} = d^{ren}\left(i,\theta^H\right)$ there is still renegotiation in the low state by assumption

[5] This is because under no credit insurance and risk-neutrality, the renegotiated amount of debt is $\frac{(\lambda-\delta)\theta^L}{2}$ as this ensures that the surplus from bargaining is shared to equal parts among the lender and the borrower. The assumption assures that $\frac{(\lambda-\delta)\theta^L}{2} > \lambda\theta^L$. Under risk-aversion or with some level of credit insurance the renegotiated amount of debt is even higher than $\frac{(\lambda-\delta)\theta^L}{2}$.

(A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i, \theta^H\right) + (1-\pi)\, d^{ren}\left(i, \theta^L\right) \quad c_H = \theta^H - d^{ren}\left(i, \theta^H\right)$$
$$c_L = (1-\delta)\,\theta^L - d^{ren}\left(i, \theta^L\right)$$

we can see that the government has a higher consumption in the high state in period 1 as it does not suffer the cost of renegotiation, while other consumption levels are the same. Thus choosing $\tilde{b} = d^{ren}\left(i, \theta^H\right)$ gives a strictly higher welfare.

<u>case 2</u> $i \in \left(\lambda\theta^L, \lambda\theta^H\right]$: under $b^* > \lambda\theta^H$ there is renegotiation in the high state while there is default in the low state

$$c_0 = w_0 + \pi d^{ren}\left(i, \theta^H\right) \quad c_H = (1-\delta)\,\theta^H - d^{ren}\left(i, \theta^H\right) \quad c_L = (1-\lambda)\,\theta^L$$

while under $\tilde{b} = d^{ren}\left(i, \theta^H\right)$ there is still default in the low state by assumption (A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i, \theta^H\right) \quad c_H = \theta^H - d^{ren}\left(i, \theta^H\right) \quad c_L = (1-\lambda)\,\theta^L$$

also in this case the government does not suffer the cost of renegotiation in the high state by choosing $\tilde{b} = d^{ren}\left(i, \theta^H\right)$ and can thus increase welfare compared to $b^* > \lambda\theta^H$

<u>case 3</u> $i > \lambda\theta^H$: under $b^* > \lambda\theta^H$ there is default in both states

$$c_0 = w_0 \quad c_H = (1-\lambda)\,\theta^H \quad c_L = (1-\lambda)\,\theta^L$$

while under $\tilde{b} = d^{ren}\left(i, \theta^H\right)$ there is still default in the low state by assumption (A1) but full repayment in the high state

$$c_0 = w_0 + \pi d^{ren}\left(i, \theta^H\right) \quad c_H = \theta^H - d^{ren}\left(i, \theta^H\right) + c_L = (1-\lambda)\,\theta^L$$

Since we have by the participation constraint of the that

$$d^{ren}\left(i, \theta^H\right) \leq (\lambda - \delta)\,\theta^H \leq \lambda\theta^H$$

consumption in the high state in period 1 is higher compared to choosing $b^* > \lambda\theta^H$ and also consumption in period 0 is higher since under this alternative choice the government repays in the high state and can thus borrow. Consumption in the low state of period 1 remains the same so that also in this last case welfare is higher under the alternative choice $\tilde{b} = d^{ren}\left(i, \theta^H\right)$. Thus $b^* > \lambda\theta^H$ cannot have been optimal. $\square$

6.2 Second best

We now proceed to show first on hand of an example and then in a general result that in equilibrium the lender chooses a (weakly) higher amount of credit insurance compared to the socially efficient level. In the following section we will compare the economy with credit insurance to the economy without credit insurance and this will also a deliver condition under which the lender strictly over-insures compared to the socially efficient level.

6.2.1 Example

Consider the economy as described above with the following parameters:

$$\theta^L = 2, \quad \theta^H = 4, \quad w_0 = 0, \quad \pi = 0.1, \quad \delta = 0, \quad \lambda = 0.8$$

In the efficient (first-best) allocation we have constant consumption across time and states

$$c^{**} = \frac{w_0 + \pi\theta^H + (1-\pi)\,\theta^L}{2} = 1.1$$

Now, if a social planner was to choose the level of credit insurance, he could implement the first best allocation by borrowing $b = 2.9 < 3.2 = \lambda\theta^H$ which would lead to full repayment in the high state and renegotiation in the low state. If he sets credit insurance to such a level i^{**} such that

$$d^{ren}\left(i^{**}, \theta^L\right) = 0.9$$

It is easy to check that this results in the efficient allocation. In equilibrium however the lender chooses the level of credit protection in order to extract the full renegotiation surplus. The renegotiated amount of debt in the low state is thus

$$d^{ren}\left(i^{*}, \theta^L\right) = (\lambda - \delta)\,\theta^L = 1.6$$

which results in the consumption level in the low state being smaller than the efficient allocation

$$c_L < c^{**}$$

The example shows that under risk-aversion it is no longer true that the lender chooses the socially efficient level of credit insurance. This is because in the incomplete contracts economy renegotiation works as implicitly adding contingency to the non-contingent bond contract. The lender however, does not internalize this effect on consumption smoothing when choosing the optimal level of credit insurance.

Proposition 8. *Under risk-aversion the lender over-insures with respect to the socially efficient choice of credit insurance, i.e.* $i^{**} \leq i^{*} = (\lambda - \delta)\,\theta^L$.

Proof. Suppose not and the socially efficient level was $i^{**} > (\lambda - \delta)\,\theta^L$.

If the government would consequently choose an amount of $b^{**} \in \left(\lambda\theta^L, \lambda\theta^H\right]$ we have seen in section 4 together with the level of credit insurance this implies default in the low state and repayment in the high state. The consumption allocation is thus

$$c_0 = w_0 + \pi b^{**} \quad c_H = \theta^H - b^{**} \quad c_L = (1-\lambda)\,\theta^L$$

If on the other hand the planner chooses an alternative level of credit insurance $\tilde{i} = (\lambda - \delta)\,\theta^L$ it is still feasible for the government to choose the same amount of debt $\tilde{b} = b^{**}$. The lower level of credit insurance implies that then there is renegotiation in the low state so that the consumption allocation is given by

$$c_0 = w_0 + \pi b^{**} + (1-\pi)\,(\lambda - \delta)\,\theta^L \quad c_H = \theta^H - b^{**} \quad c_L = (1-\lambda)\,\theta^L$$

Comparing the two expressions we can see that consumption in the initial period is higher under the alternative choice $\tilde{i} = (\lambda - \delta)\,\theta^L$ while consumption in the second period is the same. Hence $i^{**} > (\lambda - \delta)\,\theta^L$ cannot have been an optimal choice of the planner.

If on the other hand the government would choose a level of debt is low so that $b^{**} \leq \lambda\theta^L$, then there is full repayment in both states, so credit insurance does not have any effect. Thus choosing $i^{**} \leq (\lambda - \delta)\,\theta^L$ is weakly better. We have seen in the previous lemma that the government never chooses an amount of borrowing greater than $\lambda\theta^H$ which completes the proof.

□

6.3 CDS vs No-CDS

It is however not clear from the previous findings, what is the welfare effect of CDS chosen by the lender, compared to no CDS at all. Both are inefficient, so they are not trivial to compare. We will get some insight on this from the following examples.

6.3.1 Examples

Example 1

Consider the above economy with the following specifications. Utility is of CRRA form

$$u\,(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with the parameter of risk aversion taking a value of $\sigma = 5$. Other parameters are as follows: the cost of default is $\lambda = 0.2$, the cost of renegotiation is $\delta = 0$. The endowment in the high state is $\theta^H = 5$, in the low state $\theta^L = 1$. Both states are equally likely so that $\pi = 0.5$. It's a well-known result that under the assumption of risk-aversion the share of renegotiation surplus of the more risk-neutral party increases. In the case of CRRA utility it is easy to show that in the economy without credit insurance the renegotiated amount of debt is a constant proportion ρ of the bargaining surplus $d^{ren}(0,\theta^s) = \rho(\lambda-\delta)\theta^s$ where with the parameters above $\rho \approx 0.5657$ (compared to the case of risk-neutrality where $\rho = 0.5$)[6]. In the economy with credit protection on the other hand the lender buys credit insurance in order to extract the full surplus of renegotiation $\lambda\theta^L$ under any degree of risk-aversion as we have seen in section 3.3. We will first consider an economy with no initial wealth $w_0 = 0$.

Standard calculations give that in both economies the government chooses a debt level of

$$b^{*,NO} = b^{*,CDS} = \lambda\theta^H = 1$$

The government chooses thus to borrow the maximum amount possible in both economies. We thus have full repayment in the high state and renegotiation in the low state in both economies. The welfare of the government in an economy with credit insurance is

$$\begin{aligned} & u\left(\pi b^* + (1-\pi)\lambda\theta^L\right) + \pi u\left(\theta^H - b^*\right) + (1-\pi)u\left(\theta^L - \lambda\theta^L\right) \\ = \; & u(0.6) + \pi u(4) + (1-\pi)u(0.8) \\ \approx \; & -2.23 \end{aligned}$$

[6]it can be shown that ρ needs to satisfy the following non-linear equation $(1-\delta-\rho(\lambda-\delta))^{1-\sigma} - (1-\lambda)^{1-\sigma} = (1-\sigma)(1-\delta-\rho(\lambda-\delta))^{-\sigma}\rho(\lambda-\delta)$

while in the economy without credit insurance welfare is given by

$$\begin{aligned} & u\left(\pi b^{*}+(1-\pi)\,\rho\lambda\theta^{L}\right)+\pi u\left(\theta^{H}-b^{*}\right)+(1-\pi)\,u\left(\theta^{L}-\rho\lambda\theta^{L}\right) \\ \approx\; & u\,(0.5566)+\pi u\,(4)+(1-\pi)\,u\,(0.8869) \\ \approx\; & -2.81 \end{aligned}$$

We can see that welfare is higher in the economy with credit insurance. The intuition is similar to the production economy: credit insurance enables the lender to enforce a higher repayment in the low state. This leads to higher bond prices and thus higher proceeds from issuing debt. This results in a higher level of consumption in period 0 and facilitates inter-temporal consumption smoothing, which increases welfare because of the risk-aversion of the government.

Example 2

Let us now consider the similar example as in the previous section, with the only difference that now the initial wealth is given by

$$w_0 = 1$$

. Standard calculations show that the government still finds it optimal to choose

$$b^{*,NO} = b^{*,CDS} = \lambda\theta^{H} = 1$$

as constraint on borrowing is still binding. The welfare of the government in the economy with credit insurance is now

$$
\begin{aligned}
& u\left(w_0+\pi b^*+(1-\pi)\lambda\theta^L\right)+\pi u\left(\theta^H-b^*\right)+(1-\pi)\,u\left(\theta^L-\lambda\theta^L\right) \\
= \; & u\,(1.6)+\pi u\,(4)+(1-\pi)\,u\,(0.8) \\
\approx \; & -0.34
\end{aligned}
$$

while in the economy without credit insurance it is

$$
\begin{aligned}
& u\left(w_0+\pi b^*+(1-\pi)\rho\lambda\theta^L\right)+\pi u\left(\theta^H-b^*\right)+(1-\pi)\,u\left(\theta^L-\rho\lambda\theta^L\right) \\
\approx \; & u\,(1.5566)+\pi u\,(4)+(1-\pi)\,u\,(0.8869) \\
\approx \; & -0.25
\end{aligned}
$$

Observe, that as opposed to the previous example, the government would be better off in an economy without credit insurance. While increasing consumption in period 0, credit insurance also enforces a higher repayment in the low state and thus limits the amount of inter-state consumption smoothing. In this example the government values the additional proceeds from issuing debt in period 0 less, because even without credit insurance it has already a relatively high level of consumption compared to the low state. And since marginal utility is decreasing because of the concavity assumption it values additional consumption in period 0 less compared to the higher repayment in the low state so that the negative impact on inter-state consumption smoothing prevails. As we can see in the latter example the equilibrium choice of credit insurance by the lender no longer agrees with the socially efficient level. A social planner would choose the level of credit insurance taking both of the previously explained effects into account, while the lender chooses the level of credit insurance that maximizes the expected amount of repayment. The results that we found in the previous section, that credit insurance is welfare increasing and the lender always chooses the socially optimal level of credit insurance thus no longer hold under the assumption of risk-aversion, which is standard in the government debt literature.

6.3.2 General Result

We now proceed to provide sufficient conditions under which welfare in the economy without credit protection is strictly higher compared to welfare in the economy with credit protection. On one hand wealth needs to be low enough so that the government borrows an amount higher than $\lambda\theta^L$. We have seen in section 4 that otherwise credit insurance does not matter as the government repays fully in both states. The following lemma provides a condition on wealth that ensures that the optimal level of debt is higher than $\lambda\theta^L$ in the economy with credit insurance.

Lemma 9. *Suppose $\delta = 0$. If the initial wealth is low relative to the endowment in the low state so that $w_0 < (1 - 2\lambda)\,\theta^L$, in the economy with credit protection the government chooses to a debt level $b^{*,CDS} > \lambda\theta^L$ so that there is renegotiation in the low state.*

Proof. The proof proceeds by showing if the government would choose a low debt level $b^{*,CDS} \leq \lambda\theta^L$ such that there is full repayment in both states, it would find it optimal to borrow at the boundary $b^{*,CDS} = \lambda\theta^L$ and by borrowing more it can do strictly better.

We have a corner solution for low levels of debt $b^{*,CDS} \leq \lambda\theta^L$ if welfare has a strictly positive slope with respect to debt in the point $\lambda\theta^L$:

$$\begin{aligned} & u'\left(w_0+\lambda\theta^L\right)-\pi u'\left(\theta^H-\lambda\theta^L\right)-(1-\pi)\,u'\left(\theta^L-\lambda\theta^L\right) \\ > \;& u'\left(w_0+\lambda\theta^L\right)-\pi u'\left(\theta^L-\lambda\theta^L\right)-(1-\pi)\,u'\left(\theta^L-\lambda\theta^L\right) \\ = \;& u'\left(w_0+\lambda\theta^L\right)-u'\left(\theta^L-\lambda\theta^L\right)>0 \end{aligned}$$

where the last inequality follows from the assumption that $w_0 < (1-2\lambda)\,\theta^L$. We thus have to compare the welfare at higher levels of debt only with the welfare at the point $b^{*,CDS} = \lambda\theta^L$ in order to show that higher debt levels are optimal. Note that since $\delta = 0$ welfare is continuous in the point $\lambda\theta^L$ (see (6.1)). Since the derivative of welfare wrt to debt is still positive for slightly higher levels of debt $b^{*,CDS} = \lambda\theta^L + \epsilon$[7]

$$\begin{aligned} & \pi u'\left(w_0+\pi\left(\lambda\theta^L+\epsilon\right)+(1-\pi)\,\lambda\theta^L\right)-\pi u'\left(\theta^H-\left(\lambda\theta^L+\epsilon\right)\right) \\ > \;& u'\left(w_0+\lambda\theta^L+\pi\epsilon\right)-\pi u'\left(\theta^L-\lambda\theta^L+\epsilon\right) \\ > \;& 0 \end{aligned}$$

for ϵ small enough under the condition that $w_0 < (1-2\lambda)\,\theta^L$, we can increase welfare even further by choosing $b^{*,CDS} = \lambda\theta^L + \epsilon$ $\square$

In order to for credit insurance to be welfare decreasing, it needs to be the case that the cost of credit insurance - having to repay a larger amount of debt after renegotiation in the low state - outweighs the benefit - to be able to transfer a higher amount from

[7] note that we do not take the derivative with respect to the low state, as there is renegotiation for $b^{*,CDS} > \lambda\theta^L$ so that the consumption in the low state is independent of the level of debt

the low state to period 0 (and through borrowing also indirectly to the high state in period 1). The following proposition shows that this is the case if the endowment in the low state is low enough relative to a weighted average between the initial wealth and the endowment in the high state.

Proposition 10. *Suppose* $\delta = 0$*. Then for* $w_0 < (1-2\lambda)\,\theta^L$ and

$\theta^L < \min\left\{\frac{\pi\theta^H + w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H + w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$ *the economy without credit insurance strictly pareto dominates the economy with credit insurance. The lender thus strictly over-insures relative to the efficient choice of credit insurance.*

Proof. The proof proceeds in three steps.

Step 1: If the government chooses a debt level higher than $\lambda\theta^L$ both in the economy with and without credit protection we have that consumption in period 0 and in the high state in period 1 is higher in the economy with credit protection compared to the economy without credit protection, while consumption in the low state is lower in the economy with credit protection.

Proof: In what follows we define by $\rho \equiv \frac{d^{ren}\left(0,\theta^L\right)}{\lambda\theta^L}$ the share of the bargaining surplus obtained by the lender in the economy without CDS. In the economy without credit protection we then have that the level of consumption is given by

$$c_L^{NO} = (1-\rho\lambda)\,\theta^L$$

. In an interior solution $b^{*,NO} \in \left(\lambda\theta^L, \lambda\theta^H\right)$ we have that the government perfectly smoothes consumption between period 0 and the high state of period 1 so that

$$c_0^{NO} = c_H^{NO} = \frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}$$

as $\pi\theta^H + (1-\pi)\,\lambda\rho\theta^L + w_0$ is the total wealth to be shared among period 0 and the high state of period 1. At the upper bound for debt $b^{*,NO} = \lambda\theta^H$ we have that

$$c_0^{NO} = w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L$$

and

$$c_H^{NO} = \theta^H - \lambda\theta^H$$

. The optimal amount of borrowing can thus be written as

$$b^{*,NO} = \min\left\{\frac{\theta^H - (1-\pi)\,\lambda\rho\theta^L - w_0}{1+\pi}, \lambda\theta^H\right\}$$

consumption in period 0 as

$$c_0^{NO} = \min\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L\right\}$$

and consumption in the high state in period 1as

$$c_H^{NO} = \max\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\rho\theta^L}{1+\pi}, \theta^H - \lambda\theta^H\right\}$$

. Similarly we have the in the economy with credit protection where $\rho = 1$ we have that

$$c_L^{CDS} = (1-\lambda)\,\theta^L$$

$$c_0^{CDS} = \min\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\theta^L}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\theta^L\right\}$$

$$c_H^{CDS} = \max\left\{\frac{w_0 + \pi\theta^H + (1-\pi)\,\lambda\theta^L}{1+\pi}, \theta^H - \lambda\theta^H\right\}$$

and

$$b^{*,CDS} = \min\left\{\frac{\theta^H - (1-\pi)\,\lambda\theta^L - w_0}{1+\pi}, \lambda\theta^H\right\}$$

and. Comparing the expressions the result follows immediately.

Step 2: for $c_L^{NO} < c_0^{NO}$ the welfare in the economy without credit protection is strictly higher compared to the economy with credit insurance.

Proof: The intuition of this result is simple. If $c_L^{NO} < c_0^{NO}$ the government would like to shift consumption from period 0 to the low state in period 1 in the equilibrium of the economy without credit insurance. Credit insurance however works in the opposite direction as it further decreases consumption in the low state and increases consumption in period 0. Credit insurance is thus welfare decreasing. We now proceed to give the formal proof. Suppose the government borrows an amount of debt $b^{*,NO} > \lambda\theta^L$ as specified in step 1 in the economy without credit protection. The consumption allocations are then as given in step 1. The assumption $w_0 < (1-2\lambda)\,\theta^L$ together with lemma 9 ensures that the best the government can do in the economy with credit protection is to borrow $b^{*,CDS} > \lambda\theta^L$. Under these choices the results from step 1 follow immediately. By repeatedly applying a version of the intermediate value theorem[8] for

[8] the mean value theorem says that for a continuous function f and $x < y$, there exits a $\xi \in (x, y)$ s.t. $\frac{f(y)-f(x)}{y-x} = f'(\xi)$. Applying the fact that for concave functions the first derivative is decreasing

concave functions we get that

$$\begin{aligned}
& u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{NO}\right) \\
\geq\ & u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+(1-\pi)\,u'\left(c_L^{NO}\right)\left(c_L^{NO}-c_L^{CDS}\right) \\
>\ & u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+(1-\pi)\,u'\left(c_0^{NO}\right)\left(c_L^{NO}-c_L^{CDS}\right) \\
=\ & u\left(c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)+u'\left(c_0^{NO}\right)\left(\pi c_H^{CDS}-\pi c_H^{NO}+c_0^{CDS}-c_0^{NO}\right) \\
=\ & u\left(c_0^{NO}\right)+u'\left(c_0^{NO}\right)\left(c_0^{CDS}-c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+\pi u'\left(c_0^{NO}\right)\left(c_H^{CDS}-c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right) \\
\geq\ & u\left(c_0^{NO}\right)+u'\left(c_0^{NO}\right)\left(c_0^{CDS}-c_0^{NO}\right)+\pi u\left(c_H^{NO}\right)+\pi u'\left(c_H^{NO}\right)\left(c_H^{CDS}-c_H^{NO}\right)+(1-\pi)\,u\left(c_L^{CDS}\right) \\
\geq\ & u\left(c_0^{CDS}\right)+\pi u\left(c_H^{CDS}\right)+(1-\pi)\,u\left(c_L^{CDS}\right)
\end{aligned}$$

The first inequality follows from $f(y) \geq f(x)+f'(y)(y-x)$ for $y=c_L^{NO}$ and $x=c_L^{CDS}$. The strict equality in line 3 follows from the assumption that $c_L^{NO}<c_0^{NO}$ so that by concavity $u'\left(c_L^{NO}\right) \geq u'\left(c_0^{NO}\right)$ and $c_L^{NO}-c_L^{CDS} \geq 0$ as we have seen in step 1. The equality in line 3 follows from the fact that the total wealth in both economies is the same so that $c_0^{CDS}+\pi c_H^{CDS}+(1-\pi)\,c_L^{CDS}=c_0^{NO}+\pi c_H^{NO}+(1-\pi)\,c_L^{NO}$. The inequality in line 6 follows from the fact that in an interior solution we have that $c_0^{NO}=c_H^{NO}$ and if the government is constrained at $\lambda\theta^H$ $c_0^{NO}<c_H^{NO}$ so that $u'\left(c_0^{NO}\right) \geq u'\left(c_H^{NO}\right)$ and $c_H^{CDS}-c_H^{NO} \geq 0$ as we have shown step 1. The last inequality follows from applying $f(y) \leq f(x)+f'(x)(y-x)$ twice, once for $y=c_0^{CDS}$ and $x=c_0^{NO}$ and another time for $y=c_H^{CDS}$ and $x=c_H^{NO}$. We have thus shown that by choosing $b^{*,NO}$ the government can achieve a higher welfare compared to the equilibrium in the economy with credit insurance compared. Thus we have shown that the equilibrium welfare in the economy

we have that $f(y) \leq f(x)+f'(x)(y-x)$ and $f(y) \geq f(x)+f'(y)(y-x)$.

without credit insurance it higher under the conditions provided.

Step 3: if $\theta^L < \min\left\{\frac{\pi\theta^H+w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H+w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$ then $c_L^{NO} < c_0^{NO}$.

Proof: Using the expressions derived in step 1 we have that

$$c_L^{NO} = (1-\rho\lambda)\,\theta^L < \min\left\{\frac{\pi\theta^H + (1-\pi)\,\lambda\rho\theta^L + w_0}{1+\pi}, w_0 + \pi\lambda\theta^H + (1-\pi)\,\lambda\rho\theta^L\right\} = c_0^{NO}$$

After rearranging terms gives that this is equivalent to

$$\theta^L < \min\left\{\frac{\pi\theta^H + w_0}{1-2\rho\lambda+\pi}, \frac{\pi\lambda\theta^H + w_0}{1-2\rho\lambda+\pi\rho\lambda}\right\}$$

The last statement follows from the fact that we already showed in proposition 8 that the lender weakly over-insures relative to the planner's choice of credit protection. Since the welfare in the economy without credit protection is strictly higher as we have seen, the welfare in the economy with credit insurance must be lower than in the second best. □

7 Conclusion and Outlook

We have analyzed the welfare effect of credit insurance under different scenarios. We find that under risk-neutrality credit insurance is always welfare improving and the lender chooses the socially efficient level of credit insurance. This is no longer true under risk-aversion. Renegotiation implicitly adds a contingency to the non-contingent bond contract and thus enables the government to smooth consumption across states. By enforcing a higher repayment in the lower state credit insurance hinders this mechanism. In equilibrium, the lender might thus choose a level of credit protection that is higher compared to the socially efficient amount. Whether credit insurance is welfare increasing or decreasing in case of risk-aversion depends on how much the government values better terms of borrowing in period 0 compared to a lower consumption in the low state of period 1, which is implied by the endowment in the low state compared to the initial level of wealth and the level of endowment in the high state.

It would also be interesting to see what impact credit insurance has on equilibrium quantities such as bond prices and levels of debt and whether this impact also depends on the initial level of wealth. Furthermore it would be nice to have more concrete conditions under which credit insurance is welfare decreasing or increasing.

Appendix

A.1

Lemma. *For $i \leq (\lambda - \delta) f(\theta^s, k)$ the renegotiated amount of debt repayment $d^{ren}(i, \theta^s, k)$ is non-decreasing in the amount of credit insurance i*

Proof. An interior solution to the bargaining problem (3.3) must satisfy the following first order condition

$$-u'((1-\delta) f(\theta^s, k) - d^*)[d^* - i] + \quad u((1-\delta) f(\theta^s, k) - d^*) - u((1-\lambda) f(\theta^s, k)) = \quad 0$$

Applying the implicit function theorem to this equation gives

$$\begin{aligned}\frac{\partial d^*}{\partial i} &= -\frac{\overbrace{u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}^{\geq 0}}{\underbrace{u''\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)\left[d^*-i\right]}_{\leq 0}\underbrace{-u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}_{\leq 0}\underbrace{-u'\left(\left(1-\delta\right)f\left(\theta^s,k\right)-d^*\right)}_{\leq 0}} \\ &\geq 0\end{aligned}$$

so that any interior solution is increasing in i (by concavity and increasingness of the utility). A boundary solution at the upper bound is given by $d^* = (\lambda - \delta) f(\theta^s, k)$ and therefore constant with respect to i, while a boundary solution at the lower bound $d^* = i$ is trivially increasing in i. As a conclusion any solution to the bargaining problem is non-decreasing in i. □

8 References

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